Determining initial-state fluctuations from flow measurements in heavy-ion collisions

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We present a number of independent flow observables that can be measured using multiparticle azimuthal correlations in heavy-ion collisions. Some of these observables are already well known, such as $v_2\{2\}$ and $v_2\{4\}$, but most are new—in particular, joint correlations between v_1, v_2 and v_3. Taken together, these measurements will allow for a more precise determination of the medium properties than is currently possible. In particular, by taking ratios of these observables, we construct quantities which are less sensitive to the hydrodynamic response of the medium, and thus more directly characterize the initial-state fluctuations of the event shape, which may constrain models for early-time, non-equilibrium QCD dynamics. We present predictions for these ratios using two Monte-Carlo models, and compare to available data.

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INTRODUCTION

Thermalization of the matter produced in ultrarelativistic nucleus-nucleus collisions results in strong collective motion. The clearest experimental signature of collective motion is obtained from azimuthal correlations between outgoing particles. It has been recently realized [1] that fluctuations due to the internal structure of colliding nuclei (previously studied in the context of elliptic flow [2]), followed by collective flow, naturally generate specific patterns which are observed in these azimuthal correlations. In this Letter, we propose a number of independent flow measurements and study the possibility to constrain models of initial-state fluctuations directly from these experimental data.

FLOW OBSERVABLES

Correlations between particles emitted in relativistic heavy-ion collisions at large relative pseudorapidity $\Delta\eta$ are now understood as coming from collective flow [3]. According to this picture, particles in a given event are emitted independently according to some azimuthal distribution. The most general distribution can be written as a sum of Fourier components,

$$\frac{dN}{d\varphi} = \frac{N}{2\pi}\left(1 + 2\sum_{n=1}^{\infty} v_n \cos(n\varphi - n\Psi_n)\right), \qquad (1)$$

where v_n is the n^{th} flow harmonic [4] and Ψ_n the corresponding reference angle, all of which fluctuate event-by-event.

The largest flow harmonic is elliptic flow, v_2 [5], which has been extensively studied at SPS [6], RHIC [7–9], and LHC [10]. Next is triangular flow, v_3 [1], which together with v_2 is responsible for the ridge and shoulder structures observed in two-particle correlations [11, 12]. In addition, quadrangular flow, v_4, has been measured in correlation with elliptic flow [13, 14]. Finally, directed flow, v_1, can be uniquely separated [15] into a rapidity-odd part, which is the traditional directed flow [6, 13, 16], and a rapidity-even part created by initial fluctuations [17], which has only been measured indirectly [18].

In practice, one cannot exactly reconstruct the underlying probability distribution from the finite sample of particles emitted in a given event. All known information about v_n is inferred from azimuthal correlations. Generally, a k-particle correlation is of the type

$$v\{n_1, n_2, \ldots, n_k\} = \langle \cos\left(n_1\varphi_1 + \ldots + n_k\varphi_k\right)\rangle\,, \qquad (2)$$

where $n_1, \ldots, n_k$ are integers, $\varphi_1, \ldots, \varphi_k$ are azimuthal angles of particles belonging to the same event, and angular brackets denote average over multiplets of particles and events in a centrality class. Since the impact parameter orientation is uncontrolled, the only measurable correlations have azimuthal symmetry: $n_1 + \ldots + n_k = 0$.

In this work, we are interested in the global event shape. The average in Eq. (2) is thus taken over all multiplets of particles. More differential analyses (i.e., restricting one or several particles to a specific p_t interval) are left for future work. The average in Eq. (2) can be a *weighted* average, where each particle is given a weight depending on pseudorapidity and/or transverse momentum (if measured). Our goal here is to characterize initial-state fluctuations of the event shape, which are approximately independent of rapidity [19]. Weights should therefore be chosen independent of (pseudo)rapidity, which is a nonstandard choice for odd harmonics [20]. With a symmetric detector, one thus selects the rapidity-even part of v_n. We are concerned with experimental observables that can be constructed from v_1, v_2 and v_3. The study of v_4 and higher harmonics is more complicated due to the large interference with v_2 [21], and is

left for future work.

Inserting Eq. (1) into Eq. (2) gives

$$v\{n_1,\ldots,n_k\} = \langle v_{n_1}\ldots v_{n_k}\cos(n_1\Psi_{n_1}+\ldots+n_k\Psi_{n_k})\rangle\,, \tag{3}$$

where the average is now only over events. To the extent that correlations are induced by collective flow, azimuthal correlations measure moments of the flow distribution.

The simplest v_n measurement is the pair correlation [22], which corresponds to the event-averaged root-mean-square v_n

$$v_n\{2\} \equiv \sqrt{v\{n,-n\}} \simeq \sqrt{\langle {v_n}^2\rangle}. \tag{4}$$

Higher-order correlations yield higher moments of the v_n distribution:

$$v\{n,n,-n,-n\} \equiv 2v_n\{2\}^4 - v_n\{4\}^4 \simeq \langle {v_n}^4\rangle, \tag{5}$$

where we have used the standard notation $v_n\{4\}$ for the 4-particle cumulant [23].

Finally, one can construct correlations involving mixed harmonics, as in previous analyses of v_4 [13] and v_1 [24]. The first non-trivial correlations between v_1, v_2 and v_3 are

$$\begin{aligned} v_{12} &\equiv v\{1,1,-2\}, & v_{13} &\equiv v\{1,1,1,-3\}, \\ v_{23} &\equiv v\{2,2,2,-3,-3\}, & v_{123} &\equiv v\{1,2,-3\}. \end{aligned} \tag{6}$$

These observables are new. Note that $v\{1,1,-2\}$ has been analyzed with rapidity-odd weights in harmonic 1[6, 13], not with rapidity-even weights.

One generally expects $v_1 < v_3 < v_2$. Thus correlations involving high powers of v_1 are more difficult to measure. As explained in detail in Ref. [25], the analysis must be done in such a way as to isolate the correlation induced by collective flow from other "nonflow" effects, which fall into two categories: 1) global momentum conservation, whose only significant contribution is in $v\{1,-1\}$ and $v\{1,1,-2\}$. This effect can be suppressed by using the p_t-dependent weight $w = p_t - \langle p_t^2\rangle/\langle p_t\rangle$ for at least one of the particles in harmonic 1 [18]. 2) Short-range nonflow correlations, which can be suppressed by putting rapidity gaps between some of the particles. As shown in Ref. [25], all the correlations we have introduced are likely to be measurable at the LHC.

PREDICTIONS

The anisotropy in the distribution, Eq. (1), has its origin in the spatial anisotropy of the transverse density distribution at early times. Following Teaney and Yan [17], we define

$$\varepsilon_1 e^{i\Phi_1} \equiv -\frac{\{r^3 e^{i\varphi}\}}{\{r^3\}}$$

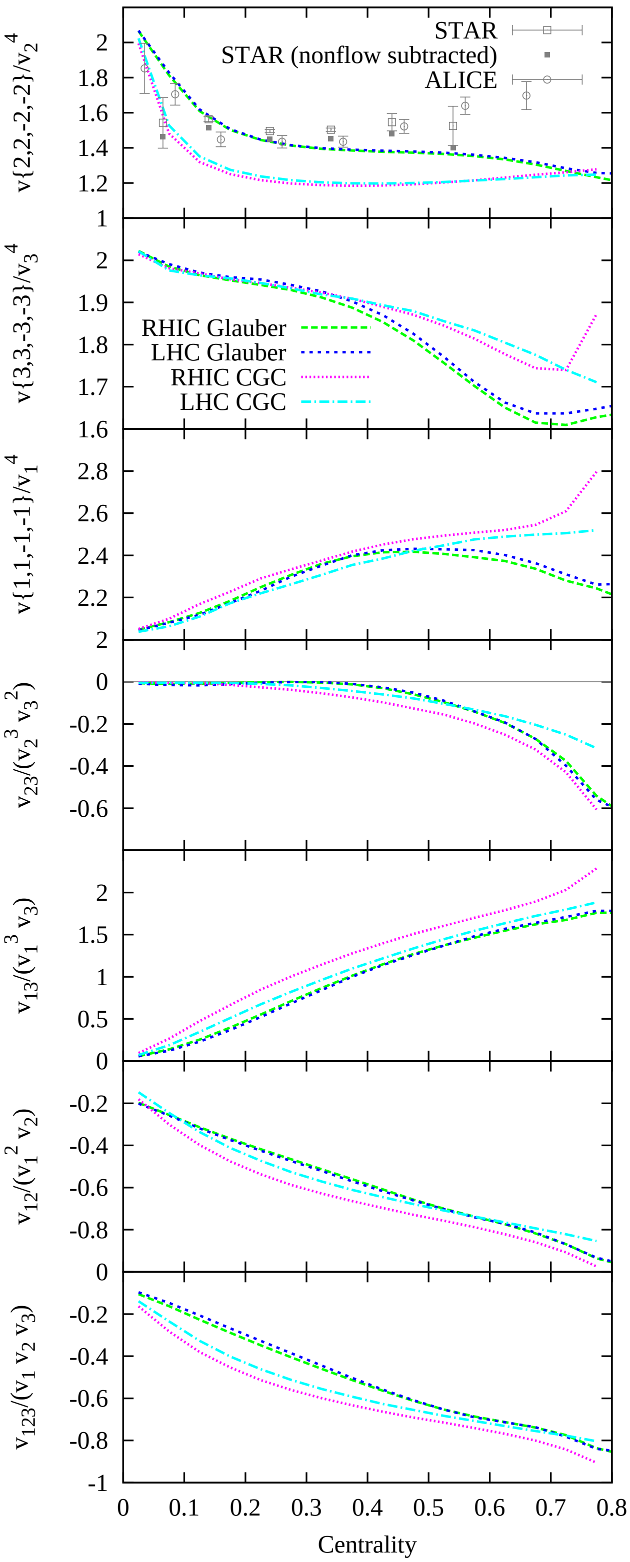

FIG. 1. (Color online) Predictions for ratios of various proposed measurements as a function of centrality (fraction of the total cross section, most central to the left) in Au-Au collisions at RHIC and Pb-Pb collisions at LHC, using a Glauber- and a CGC-type model with 100 million and 20 million events, respectively. The factors in the denominator are shorthand, $v_n \equiv v_n\{2\}$. See text for details.

$$\varepsilon_2 e^{2i\Phi_2} \equiv -\frac{\{r^2 e^{2i\varphi}\}}{\{r^2\}}$$
$$\varepsilon_3 e^{3i\Phi_3} \equiv -\frac{\{r^3 e^{3i\varphi}\}}{\{r^3\}}, \qquad (7)$$

where curly brackets denote an average over the transverse plane in a single event [26], weighted by the density at midrapidity, and the distribution is centered in each event, $\{re^{i\varphi}\} = 0$. In this equation, Φ_n is the minor orientation angle (corresponding, e.g., to the minor axis of the ellipse for n=2), and ε_n the magnitude of the respective anisotropy.

Anisotropic flow scales like the initial anisotropy ε_n and develops along Φ_n. It is therefore natural to expect that at a given centrality, $v_n = K_n\varepsilon_n$ and $\Psi_n = \Phi_n$, where K_n is a constant that contains all information about the hydrodynamic response to the initial anisotropy in harmonic n — in particular, medium properties such as the equation of state, viscosity, etc. These relations are not exact, but event-by-event hydrodynamic calculations have shown that they indeed hold to a good approximation for v_1 [15], v_2 [27, 28] and v_3 [28, 29] — in the majority of events, the angles $\Psi_n \simeq \Phi_n$ are strongly correlated, while v_n/ε_n is close to a constant value for a given set of parameters. There are typically only small and apparently random deviations on an event-by-event basis, making the use of this approximation very useful for event-averaged quantities such as those considered here. The validity of these relations goes beyond hydrodynamics, and they still hold if the system is far from equilibrium [30]. (On the other hand, these simple relations are not valid for higher harmonics such as v_4 and v_5 [28], which is why we focus here on $n \leq 3$.)

Inserting these proportionality relations into Eq. (3), we obtain

$$v\{n_1, \ldots, n_k\} = K_{n_1} \ldots K_{n_k} \varepsilon\{n_1, \ldots, n_k\}, \qquad (8)$$

where we have introduced the notation

$$\varepsilon\{n_1, \ldots, n_k\} \equiv \langle \varepsilon_{n_1} \ldots \varepsilon_{n_k} \cos(n_1\Phi_{n_1} + \ldots + n_k\Phi_{n_k})\rangle . \qquad (9)$$

Thus the measured correlations are sensitive to details of the hydrodynamic evolution mostly through the coefficients K_n, and to the initial-state dynamics through $\varepsilon\{n_1, \ldots, n_k\}$, which only contains information about the system prior to equilibration.

At present, properties of the early-time system are poorly constrained by experiment, and contribute the largest source of uncertainty in the extraction of medium properties like shear viscosity [31]. However, these new proposed flow measurements will now provide an opportunity to significantly constrain the initial state.

In particular, one can eliminate the dependence on the proportionality coefficients K_n —and therefore isolate initial-state effects— by measuring the correlations defined by Eq. (2), integrated over phase space, and by scaling them appropriately:

$$\frac{v\{n_1, n_2, \ldots, n_k\}}{v_{n_1}\{2\} \ldots v_{n_k}\{2\}} = \frac{\varepsilon\{n_1, n_2, \ldots, n_k\}}{\varepsilon_{n_1}\{2\} \ldots \varepsilon_{n_k}\{2\}}, \qquad (10)$$

where $\varepsilon_n\{2\} \equiv \sqrt{\langle \varepsilon_n{}^2 \rangle}$. The left-hand side of Eq. (10) can be measured experimentally, while the right-hand side depends only on early-time dynamics, and can be calculated using a model of initial-state fluctuations. Thus, although the relations (8) are only approximate, taking these ratios minimizes sensitivity to medium properties and — even though they come from correlations between soft particles — they represent some of the most direct probes of initial-state dynamics available.

Eq. (10) holds if the coefficients K_n are positive. With the definitions Eq. (7), this always holds for K_2 and K_3. However, K_1 is negative for low p_t particles [15, 17]. One can compensate for this negative sign by giving a negative weight to low p_t particles in Eq. (2) [18].

We make predictions for these ratios by computing them with two of the most common models for the initial state of a heavy-ion collision. First is the PHOBOS Glauber Monte-Carlo [32], with binary collision fraction $x = 0.145$ for RHIC collisions and $x = 0.18$ for LHC. The second uses the gluon density from a color-glass-condensate (CGC) inspired model — the MC-KLN [33], with rcBK unintegrated gluon densities [34]. The main difference between the two models is that the eccentricity is larger in the CGC model [35, 36]. Both models are fairly simple, with the only source of fluctuations being the nucleonic structure of nuclei. In reality other sources of fluctuations could be important, and future study will be needed to fully understand the constraints imposed on the initial dynamics by these measurements.

Fig. 1 displays predictions for all of the scaled correlations in Au-Au collisions at 200 GeV per nucleon pair and Pb-Pb collisions at 2.76 TeV per nucleon pair.

The top three panels show $v\{n, n, -n, -n\}/v_n\{2\}^4 = \langle v_n{}^4 \rangle / \langle v_n{}^2 \rangle^2$. For Gaussian fluctuations [37], the n=1 and 3 ratios are equal to 2 (i.e., $v_n\{4\}$=0), and likewise for n=2 in central collisions. However, this is expected only in the limit of a large system. A more detailed analysis [38] shows that, e.g., $v_3\{4\}$ should be smaller than $v_3\{2\}$ only by a factor $\sim$2 in mid-central collisions, in agreement with these results. Note that wherever the ratio is greater than 2, the fourth cumulant $v_n\{4\}$ is undefined. The top panel also shows existing data from STAR [39] and ALICE [10]. Neither measurement includes a rapidity gap, and thus may contain nonflow correlations (see the discussion in Ref. [25]). For STAR $v_2\{2\}$, we use both the raw data, and the value with an estimated correction for nonflow effects [40]. The data seem to favor larger relative fluctuations than are contained in the MC-KLN model used here.

The bottom four panels display scaled mixed correlations, indicating non-trivial correlations between Ψ_1, Ψ_2

and Ψ_3. The scaled correlation v_{23} indicates a negligible correlation between Ψ_2 and Ψ_3 up to 40% centrality in the Glauber model [1, 41], while the CGC model predicts a small anticorrelation. In contrast, Ψ_1 has both a strong correlation with Ψ_3 [42] (positive v_{13}) and a (weaker) anticorrelation with Ψ_2 [17] (negative v_{12}), though this decreases for central collisions. The dependence on impact parameter can be attributed to the intrinsic eccentricity of the nuclear overlap region [38]. The strong positive correlation between Ψ_1 and Ψ_3 explains why v_{12} and v_{123} [17] have the same sign and behave similarly.

CONCLUSION

We have proposed a new set of independent flow observables in heavy-ion collisions which can be combined to tightly constrain theoretical models. In particular, certain ratios are constructed which are largely determined only by the initial state, and thus directly measure properties of the early-time system. We have presented predictions for these ratios using two common Monte-Carlo models, and compared to existing data.

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[1] B. Alver and G. Roland, Phys. Rev. C **81**, 054905 (2010) [Erratum-ibid. C **82**, 039903 (2010)] [arXiv:1003.0194 [nucl-th]].
[2] B. Alver *et al.* [PHOBOS Collaboration], Phys. Rev. Lett. **98**, 242302 (2007) [arXiv:nucl-ex/0610037].
[3] M. Luzum, Phys. Lett. B **696**, 499 (2011) [arXiv:1011.5773 [nucl-th]].
[4] S. Voloshin and Y. Zhang, Z. Phys. C **70**, 665 (1996) [arXiv:hep-ph/9407282].
[5] J.-Y. Ollitrault, Phys. Rev. D **46**, 229 (1992).
[6] C. Alt *et al.* [NA49 Collaboration], Phys. Rev. C **68**, 034903 (2003) [arXiv:nucl-ex/0303001].
[7] K. H. Ackermann *et al.* [STAR Collaboration], Phys. Rev. Lett. **86**, 402 (2001) [arXiv:nucl-ex/0009011].
[8] S. S. Adler *et al.* [PHENIX Collaboration], Phys. Rev. Lett. **91**, 182301 (2003) [arXiv:nucl-ex/0305013].
[9] B. B. Back *et al.* [PHOBOS Collaboration], Phys. Rev. C **72**, 051901 (2005) [arXiv:nucl-ex/0407012].
[10] K. Aamodt *et al.* [The ALICE Collaboration], Phys. Rev. Lett. **105**, 252302 (2010). [arXiv:1011.3914 [nucl-ex]].
[11] A. Adare *et al.* [PHENIX Collaboration], Phys. Rev. C **78**, 014901 (2008) [arXiv:0801.4545 [nucl-ex]].
[12] J. Adams *et al.* [Star Collaboration], Phys. Rev. C **75**, 034901 (2007) [arXiv:nucl-ex/0607003].
[13] J. Adams *et al.* [STAR Collaboration], Phys. Rev. Lett. **92**, 062301 (2004) [arXiv:nucl-ex/0310029].
[14] A. Adare *et al.* [PHENIX Collaboration], Phys. Rev. Lett. **105**, 062301 (2010) [arXiv:1003.5586 [nucl-ex]].
[15] F. G. Gardim, F. Grassi, Y. Hama, M. Luzum and J.-Y. Ollitrault, Phys. Rev. C **83**, 064901 (2011) [arXiv:1103.4605 [nucl-th]].
[16] B. B. Back *et al.* [PHOBOS Collaboration], Phys. Rev. Lett. **97**, 012301 (2006) [arXiv:nucl-ex/0511045].
[17] D. Teaney and L. Yan, Phys. Rev. C **83**, 064904 (2011) [arXiv:1010.1876 [nucl-th]].
[18] M. Luzum and J.-Y. Ollitrault, Phys. Rev. Lett. **106**, 102301 (2011) [arXiv:1011.6361 [nucl-ex]].
[19] A. Dumitru, F. Gelis, L. McLerran and R. Venugopalan, Nucl. Phys. A **810**, 91 (2008) [arXiv:0804.3858 [hep-ph]].
[20] A. M. Poskanzer and S. A. Voloshin, Phys. Rev. C **58**, 1671 (1998) [arXiv:nucl-ex/9805001].
[21] N. Borghini and J.-Y. Ollitrault, Phys. Lett. B **642**, 227 (2006) [arXiv:nucl-th/0506045].
[22] S. Wang *et al.*, Phys. Rev. C **44**, 1091 (1991).
[23] N. Borghini, P. M. Dinh and J.-Y. Ollitrault, Phys. Rev. C **64**, 054901 (2001) [arXiv:nucl-th/0105040].
[24] N. Borghini, P. M. Dinh and J.-Y. Ollitrault, Phys. Rev. C **66**, 014905 (2002) [arXiv:nucl-th/0204017].
[25] R. S. Bhalerao, M. Luzum and J. Y. Ollitrault, J. Phys. G **38**, 124055 (2011) [arXiv:1106.4940 [nucl-ex]].
[26] R. S. Bhalerao and J.-Y. Ollitrault, Phys. Lett. B **641**, 260 (2006) [arXiv:nucl-th/0607009].
[27] H. Holopainen, H. Niemi and K. J. Eskola, Phys. Rev. C **83**, 034901 (2011) [arXiv:1007.0368 [hep-ph]].
[28] Z. Qiu and U. W. Heinz, Phys. Rev. C **84**, 024911 (2011) [arXiv:1104.0650 [nucl-th]].
[29] G. Y. Qin, H. Petersen, S. A. Bass and B. Muller, Phys. Rev. C **82**, 064903 (2010) [arXiv:1009.1847 [nucl-th]].
[30] N. Borghini and C. Gombeaud, Eur. Phys. J. C **71**, 1612 (2011) [arXiv:1012.0899 [nucl-th]].
[31] M. Luzum and P. Romatschke, Phys. Rev. C **78**, 034915 (2008) [Erratum-ibid. C **79**, 039903 (2009)] [arXiv:0804.4015 [nucl-th]].
[32] B. Alver, M. Baker, C. Loizides and P. Steinberg, arXiv:0805.4411 [nucl-ex].
[33] H. J. Drescher and Y. Nara, Phys. Rev. C **76**, 041903 (2007) [arXiv:0707.0249 [nucl-th]].
[34] J. L. Albacete and A. Dumitru, arXiv:1011.5161 [hep-ph].
[35] T. Hirano, U. W. Heinz, D. Kharzeev, R. Lacey and Y. Nara, Phys. Lett. B **636**, 299 (2006) [arXiv:nucl-th/0511046].
[36] T. Lappi and R. Venugopalan, Phys. Rev. C **74**, 054905 (2006) [arXiv:nucl-th/0609021].
[37] S. A. Voloshin, A. M. Poskanzer, A. Tang and G. Wang, Phys. Lett. B **659**, 537 (2008) [arXiv:0708.0800 [nucl-th]].
[38] R. S. Bhalerao, M. Luzum and J. -Y. Ollitrault, Phys. Rev. C **84**, 054901 (2011) [arXiv:1107.5485 [nucl-th]].
[39] J. Adams *et al.* [STAR Collaboration], Phys. Rev. C **72**, 014904 (2005) [arXiv:nucl-ex/0409033].
[40] J.-Y. Ollitrault, A. M. Poskanzer and S. A. Voloshin, Phys. Rev. C **80**, 014904 (2009) [arXiv:0904.2315 [nucl-ex]].
[41] J. L. Nagle, M. P. McCumber, Phys. Rev. **C83**, 044908 (2011). [arXiv:1011.1853 [nucl-ex]].
[42] P. Staig and E. Shuryak, Phys. Rev. C **84**, 034908 (2011) [arXiv:1008.3139 [nucl-th]].